Remarks by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

In Salmon's *Conic Sections*, Sixth Edition, Articles 188-189, it is demonstrated that if through any point P of a conic section we draw tangents PT, QT to a confocal conic section, these tangents are equally inclined to the tangent at P. For every point P the outer conic through P, though confocal to the inner, has different axes.

If we wish to find a curve for all positions of P the problem assumes some difficulty.

For the ellipse, the equation to TP, TQ is

$$(a^2-h^2)(y-k)^2+2(y-k)(x-h)hk+(b^2-k^2)(x-h)^2=0,$$

where (h, k) are the coordinates of T.

$$2hk(y-k)+\{b^2-a^2+h^2-k^2\pm\sqrt{[(b^2-a^2+h^2-k^2)^2+4h^2k^2]}\}(x-h)=0,$$

gives the two lines making equal angles with TP, TQ.

The envelope of the line formed by using the plus sign subject to the condition obtained by using the minus sign gives the required curve.

If $a=b$, the ellipse becomes a circle. Then TT' is

$$ky+hx=h^2+k^2\ldots(1).$$

$hy=kx$ is the perpendicular to (1).

The envelope of (1) subject to the condition $hy=kx$,is $x^2+y^2=0$, or the center of the given circle.

266. Proposed by C. N. SCHMALL, New York City.

Show that the nth derivative of the fraction u/v can be expressed in the form of a determinant, u and v being functions of x.

Solution by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

As $u=\phi(x)$, $v=\psi(x)$, hence, $F(x)=\phi(x)/\psi(x)=u/v$, and therefore,

$$F'(x)=\frac{vu'-uv'}{v^2}=\frac{1}{v^2}\begin{vmatrix} v & v' \\ u & u' \end{vmatrix}=\frac{u_1}{v_1}=\frac{\phi_1(x)}{\psi_1(x)}.$$

Likewise, $F''(x)=\dfrac{1}{v_1^2}\begin{vmatrix} v_1 & v'_1 \\ u_1 & u'_1 \end{vmatrix}$; $F'''(x)=\dfrac{1}{v_2^2}\begin{vmatrix} v_2 & v'_2 \\ u_2 & u'_2 \end{vmatrix}$; ...

$$\therefore F^{(n)}(x)=\frac{1}{v^2_{n-1}}\begin{vmatrix} v_{n-1} & v'_{n-1} \\ u_{n-1} & u'_{n-1} \end{vmatrix},\text{ where } v_\lambda=v^{2^\lambda},\quad v'_\lambda=2^\lambda v^{2^\lambda-1}v';$$

$$u_\lambda=\begin{vmatrix} v_{\lambda-1} & v'_{\lambda-1} \\ u_{\lambda-1} & u'_{\lambda-1}\end{vmatrix}, \text{ and } u'_\lambda=\begin{vmatrix} v_{\lambda-1} & v''_{\lambda-1} \\ u_{\lambda-1} & u''_{\lambda-1}\end{vmatrix}\cdots$$

Also solved by G. B. M. Zerr.

267. Proposed by FRANK LOXLEY GRIFFIN, S. M., Ph. D., Instructor in Mathematics, Williams College.

A point within an ellipse, upon a normal making an angle λ with the major axis, is arbitrarily chosen. With this point as pole, and the line through it parallel to the major axis as polar axis, the equation of the ellipse is, $A\cos^4\theta+B\cos^3\theta+C\cos^2\theta+D\cos\theta+E=0$, where the coefficients are functions of λ, of the radius vector ρ, and of the distance along the normal to the pole, ρ_1. Evidently for $\rho=\rho_1$, a solution is $\cos\theta=\cos\lambda$. Required the multiplicity of this solution for any values of ρ_1, $[\lambda\neq0,\ \rho_1\neq0]$.

Solution by the PROPOSER.

I. *Analytical Solution.* The normal to $\dfrac{x^2}{a^2}+\dfrac{y^2}{a^2(1-e^2)}=1$ at (x_0, y_0) is, denoting its slope by $\tan\lambda$, $\dfrac{x_0-\xi}{\cos\lambda}=\dfrac{y_0-\eta}{\sin\lambda}=r$, where r is the distance from (x_0, y_0) to any point (ξ, η), being positive for interior points. Now, $\tan\lambda=-\dfrac{dx_0}{dy_0}=\dfrac{x_0}{x_0(1-e^2)}$, whence $x_0^2+x_0^2(1-e^2)\tan^2\lambda=a^2$, or

$$x_0=\mu\cos\lambda,\ y_0=\mu\sin\lambda(1-e^2),\ \mu=\frac{a}{\sqrt{(1-e^2\sin^2\lambda)}}. \tag{1}$$

And, if the pole (x_1, y_1):

$$x_1=x_0-\rho_1\cos\lambda,\ y_1=y_0-\rho_1\sin\lambda. \tag{2}$$

Transfer the origin to (x_1, y_1) and introduce polar coordinates by $x=x_1+\rho\cos\theta$, $y=y_1+\rho\sin\theta$; the ellipse becomes

$$(1-e^2)(x_1+\rho\cos\theta)^2+(y_1+\rho\sin\theta)^2=a^2(1-e^2). \tag{3}$$

Collect, and eliminate $\sin\theta$:

$$(\alpha\cos^2\theta+\beta\cos\theta+\gamma)^2=(-\delta\sin\theta)^2=\delta^2(1-\cos^2\theta), \tag{4}$$

$$\text{where } \alpha=-e^2\rho^2,\ \beta=2x_1\rho(1-e^2),\ \delta=2y_1\rho, \text{ and } \gamma=\rho^2+(x_1^2-a^2)(1-e^2)+y_1^2. \tag{5}$$

The re-arrangement of (4) in descending powers of $\cos\theta$ gives the form of equation used in stating the problem; the above form will, however, suffice.